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ELECTRIFICATION OF AEROSOL PARTICLES MOVING IN A ONE-DIMENSIONAL CORONAL DISCHARGE

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The electrification of dispersed aerosol particles as the latter move through the region of a one-dimensional, unipolar coronal discharge is analyzed in the approximation of electrohydrodynamics [1, 2]. The problem of such a discharge in a stationary gas was solved in [3], while it was solved for gas motion at a constant velocity in [4]. A numerical investigation on a computer of the problem of a one-dimensional coronal discharge in an aerosol and the charging of its particles in the case when the aerosol moves in the direction of ion motion was carried out in [4], where the influence of the charging of aerosol particles on the coronal discharge is also taken into account.

In the present work we consider cases when the aerosol moves in the direction of ion motion or opposite to it, while the aerosol particles do not affect the coronal discharge. An exact analytical solution of the problem of particle charging is found in this formulation, it is investigated, and simple asymptotic expressions are obtained for the dependence of the particle charge on the local value of the electric field strength and the aerosol velocity.

1. Let us consider one-dimensional steady flow of an aerosol consisting of a gas and initially uncharged, dispersed liquid particles through the region of a unipolar coronal discharge between two plane grid electrodes placed perpendicular to the stream. For determinacy we assume that the collector electrode is grounded (we take its potential as zero), while to create the coronal discharge a system of needles, which start to display corona at an emitter potential Φ_0, is installed on the emitter electrode. Let the distance L between the collector and the emitter be sufficiently large and let nonuniformity of the electric field near the grid electrodes be neglected. We choose the Cartesian coordinate system x, y, z so that the emitter and collector lie in the planes x = 0 and x = L. We are confined to the case when the influence of the electric field on the motion of the gas and aerosol particles is small. For this it is sufficient to satisfy the inequalities

$$|qE|L/\rho u^2 \ll 1, \quad qbE^2L/(\rho c_V T|u|) \ll 1,$$
$$\min\left(|QE|/(6\pi\mu a|u|), \quad |QE|L/mu^2\right) \ll 1,$$

where q is the electric charge density of ions in the region of the unipolar coronal discharge; b is their mobility (b > 0 for a positive coronal discharge while b < 0 for a negative one); u and E are the projections of the gas velocity and the electric field strength onto the x axis; ρ, μ, c_V, and T are the density, viscosity, specific heat, and temperature of the gas (its relative permittivity is taken as one); Q, m, and a are the electric charge,

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mass, and radius of an aerosol particle. To derive these inequalities we must estimate the relative orders of magnitude of the terms characterizing the influence of the electric field in the equations of momentum and heat inflow for a unipolarly charged gas and in the equation of motion of charged aerosol particles [2] and write the conditions for their smallness. As a result, we obtain the above inequalities, which have the following meaning. The first two inequalities mean that the work of Coulomb forces acting on the gas is much less than its kinetic energy, while the inflow of Joule heat to the gas is small compared with its internal energy, and hence the influence of the electric field on the gas motion is insignificant. The last inequality means that the characteristic drift velocity of aerosol particles relative to the gas under the action of Coulomb forces is small compared with the gas velocity or that the work of these forces is small compared with the initial kinetic energy of the particles, and hence the influence of the electric field on their motion is insignificant. Then the velocities of the gas and the aerosol particles can be considered as constant and equal to each other. Also let the concentration n of aerosol particles be sufficiently low ($n|Q| \ll |q|$) and let their influence on the values of E and q be neglected.

As a result, the differential equations and boundary conditions describing the coronal discharge and the charging of aerosol particles while moving in it will have the form

$$dE/dx = 4\pi q, \quad E = -d\varphi/dx, \quad q(u + bE) = j = \text{const}; \tag{1.1}$$

$$\varphi(0) = \Phi, \quad E(0) = \Phi_0/L \equiv E_0, \quad \varphi(L) = 0; \tag{1.2}$$

$$u\frac{dQ}{dx} = J, \quad J = \begin{cases} 3\pi a^2 bqE\left(1 - \dfrac{Q}{3Ea^2}\right)^2, & \dfrac{Q}{3Ea^2} \leqslant 1, \\ 0, & \dfrac{Q}{3Ea^2} \geqslant 1; \end{cases} \tag{1.3}$$

$$Q(0) = 0, \quad u > 0; \quad Q(L) = 0, \quad u < 0. \tag{1.4}$$

Here φ is the electric potential; j, electric current density of ions; Φ, emitter potential ($|\Phi| > |\Phi_0|$); J, electric current flowing to an aerosol particle of radius a due to the capture of ions by it under the action of the electric field. The constant quantity u appearing in Eqs. (1.2) and (1.4) can be both larger and smaller than zero, and for $u < 0$ it is assumed that $u + bE > 0$ everywhere in the interelectrode gap. For $u > 0$ this inequality is automatically satisfied, since $bE > 0$ follows from the statement of the problem. The second equality of (1.2) means that after the ignition of the coronal discharge the electric field strength near the emitter remains constant with an increase in $|\Phi|$ [5]. The derivation of Eq. (1.3) for J was discussed in [6, 7] under the assumption that the conductivity of the aerosol particles is much greater than the conductivity of the gas. Equations (1.1)-(1.4) can be used for both a positive and a negative coronal discharge. In the latter case $\Phi < \Phi_0 < 0$, $E < 0$, $q < 0$, $b < 0$, and $j < 0$.

Integrating Eqs. (1.1) with allowance for the first two boundary conditions of (1.2), we obtain

$$E = b^{-1}(\sqrt{(u + bE_0)^2 + 8\pi bjx} - u); \tag{1.5}$$

$$\varphi = \Phi + \frac{ux}{b} - \frac{1}{12\pi b^2 j}\left[((u + bE_0)^2 + 8\pi bjx)^{3/2} - (u + bE_0)^3\right]. \tag{1.6}$$

From Eqs. (1.5) and (1.6), using the last condition (1.2), we find the electric field strength $E_L(\Phi)$ at the end of the interelectrode gap and the electric current density $j(E_L)$ which appears in (1.5) and (1.6):

$$E_L = \frac{1}{b}\left[\frac{3}{4}\left(\frac{b\Phi}{L} - u\right) - \frac{bE_0}{2} + \sqrt{\frac{9}{16}\left(\frac{b\Phi}{L} + u\right)^2 + \frac{3}{4}(u + bE_0)\left(\frac{b\Phi}{L} - bE_0\right)}\right],$$

$$j = \frac{1}{8\pi bL}\left[(u + bE_L)^2 - (u + bE_0)^2\right].$$

2. Let us investigate the charging of aerosol particles when the aerosol moves through the discharge gap in the direction from the emitter toward the collector ($u > 0$). In this case the absolute value of the electric field strength grows monotonically in the direction of motion of the aerosol particles. As a result, the absolute values of the charges of the aerosol particles grow as they move in the entire interelectrode gap.

Choosing the quantity E as the independent variable instead of x and using the first equation of (1.1), we reduce the equation of charging of an aerosol particle (1.3) to an

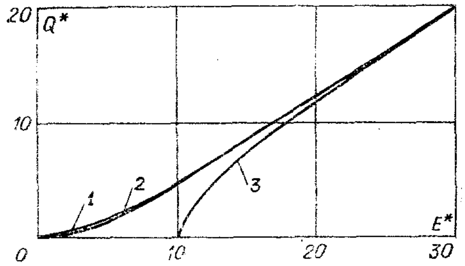

Fig. 1.

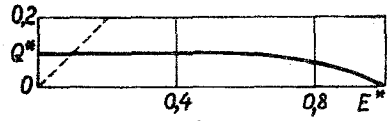

Fig. 2.

equation of the Riccati type,

$$dw/dE + (w - 1)/E + (b/4u)w^2 = 0, \quad w \equiv (1 - Q/3Ea^2). \tag{2.1}$$

By the substitution $w(E) = 2\eta'(\xi)/\xi\eta(\xi)$, $\xi = \sqrt{bE/u}$, Eq. (2.1) is reduced to a zeroth-order modified Bessel equation for the function $\eta(\xi)$. Solving the latter and using the boundary condition (1.4), we find the following expression for $Q(E)$:

$$Q = (3a^2u/b)\,Q^*(E^*, E_0^*), \quad E^* \equiv bE/u, \quad E_0^* \equiv bE_0/u; \tag{2.2}$$

$$Q^* = E^* \frac{K_2\left(\sqrt{E_0^*}\right) I_2(\sqrt{E^*}) - I_2\left(\sqrt{E_0^*}\right) K_2(\sqrt{E^*})}{K_2\left(\sqrt{E_0^*}\right) I_0(\sqrt{E^*}) - I_2\left(\sqrt{E_0^*}\right) K_0(\sqrt{E^*})}. \tag{2.3}$$

In the particular case when $E_0^* = 0$ Eq. (2.3) is simplified:

$$Q^* = E^* I_2(\sqrt{E^*})/I_0(\sqrt{E^*}).$$

In Fig. 1 we present the dependence $Q^*(E^*)$ for $E_0^* = 0$, 1, and 10 (curves 1-3, respectively). It is seen that the charge of an aerosol particle grows monotonically with an increase in electric field strength and for $E^* \gtrsim 25$ it hardly depends on the electric field strength at the start of the interelectrode gap if $E_0^* \lesssim 10$.

Using an asymptotic expansion of the modified Bessel functions for large and small values of the arguments [8], one can show that

$$Q^* = (\sqrt{E^*} - 1)^2 + O(1/\sqrt{E^*}), \quad \sqrt{E^*} - \sqrt{E_0^*} > \text{const}, \quad E^* \to \infty; \tag{2.4}$$

$$Q^* = (1/8)(E^{*2} - E_0^{*2}) + O(E^{*3}), \quad E^* \to 0, \quad E_0^* \to 0. \tag{2.5}$$

It follows from Fig. 1 that the quantity $O(1/\sqrt{E^*})$ in Eq. (2.4) is small for $E^* \geqslant 25$ and $E_0^* \leqslant 10$. Equations (2.4) formally correspond, for example, to the cases of $E \to \infty$ (E_0 = const, u = const) or $u \to 0$ (E = const, E_0 = const) while Eqs. (2.5) correspond to the case of $u \to \infty$ (E = const, E_0 = const).

3. Now let us investigate the charging of aerosol particles when the aerosol moves through the discharge gap in the direction from the collector toward the emitter ($u < 0$). In this case the absolute value of the electric field strength decreases monotonically in the direction of motion of the aerosol particles. As a result, we have

$$udQ/dx = J \neq 0, \quad x_c < x < L, \tag{3.1}$$
$$udQ/dx = J = 0, \quad 0 < x \leqslant x_c,$$

where the x coordinate is found from the equation

$$Q(x_c) = 3a^2E(x_c) \tag{3.2}$$

and, as will be shown below, does not depend on the particle radius a.

According to Eqs. (3.1), the absolute values of the charges of the aerosol particles grow as they move through the region of $x_c < x < L$ adjacent to the collector. But in the region of $0 < x \leqslant x_c$ adjacent to the emitter the charges of the aerosol particles remain constant. If Eq. (3.2) has no solution x_c out of the interval $(0, L)$, then the absolute values of the charges of the aerosol particles grow as they move in the entire interelectrode gap.

As in Sec. 2, we reduce the equation of the charging of an aerosol particle (1.3) to an equation of the Riccati type (in the region of $|E| > |E_c| \equiv |E(x_c)|$ adjacent to the collector):

$$dw/dE + (w - 1)/E - (b/4|u|)w^2 = 0, \quad w \equiv (1 - Q/3Ea^2). \tag{3.3}$$

By the substitution $w(E) = -2\eta'(\xi)/\xi\eta(\xi)$, $\xi = \sqrt{bE/|u|}$, Eq. (3.3) is reduced to a zeroth-order Bessel equation for the function $\eta(\xi)$. Solving the latter and using the boundary condition (1.4), we find the following expression for Q(E):

$$Q = (3a^2|u|/b)\,Q^*(E^*, E_L^*), \quad E^* = bE/|u|, \quad E_L^* = bE_L/|u|; \tag{3.4}$$

$$Q^* = -E^*\frac{Y_2\left(\sqrt{E_L^*}\right)J_2(\sqrt{E^*}) - J_2\left(\sqrt{E_L^*}\right)Y_2(\sqrt{E^*})}{Y_2\left(\sqrt{E_L^*}\right)J_0(\sqrt{E^*}) - J_2\left(\sqrt{E_L^*}\right)Y_0(\sqrt{E^*})}. \tag{3.5}$$

Equation (3.5) for Q^* is valid only for $E^* \geqq E_c^*$. Obviously, for $E^* < E_c^*$ we have $Q^* = Q^*(E_c^*, E_L^*) = \text{const}$. The value of E_c^* is found from the equation

$$Q^*(E_c^*, E_L^*) = E_c^*, \tag{3.6}$$

which follows from the condition (3.2) and the expressions (3.4) for Q and E^*. From Eq. (3.6) it follows that the value of E_c^* does not depend on the radius of the aerosol particle, and hence the coordinate x_c also does not depend on it. If $E_0^* < E_c^*$, then the value of the electric field strength E_c at which the current J is reduced to zero is reached within the interelectrode gap at the point x_c.

The function $Q^*(E^*)$ for $E_L^* = 1$ is presented in Fig. 2. It is seen that for a value of $E^* = 0.6$ the quantity Q^* is already close to its maximum value of 0.1, which is reached for $E^* = E_c^* = 0.1$.

Using asymptotic expansions of the Bessel functions at large values of the arguments [8], we can show that for $E_L^* > E^* \to \infty$ and $\theta \equiv \sqrt{E_L^*} - \sqrt{E^*} > \delta$ we have

$$\begin{gathered} Q^* = (\sqrt{E^*} - \operatorname{ctg}\theta)^2 + 3/\sin^2\theta + O(1/\sqrt{E^*}), \quad E^* > E_c^*, \\ Q^* = E_c^* = \left(\sqrt{E_L^*} - \pi/2\right)^2 + 3 + O\left(1/\sqrt{E_L^*}\right), \quad E^* \leqslant E_c^*. \end{gathered} \tag{3.7}$$

Here $0 < \delta < \theta_c = \sqrt{E_L^*} - \sqrt{E_c^*} = \pi/2 - 3/(2\sqrt{E_L^*}) + O(1/E_L^*)$.

Equations (3.7) formally correspond, for example, to the cases of $E_L \to \infty$ (u = const) or $u \to 0$ (E_L = const), with $E_c \to E_L$ and $x_c \to L$ in the latter case, obviously.

The equations for Q(E) obtained in Secs. 2 and 3, with allowance for the expression (1.5) for E(x), give the function Q(x) in the problem under consideration.

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